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"On the Calculation of the Coefficient of Mutual Induction of a Circle and a Coaxial Helix, and of the Electromagnetic Force between a Helical Current and a Uniform Coaxial Circular Cylindrical Current Sheet." By J. VIRIAMU JONES, F.R.S. Received November 12,—Read December 9, 1897.

(Abstract.)

§ 1. Let M_Θ be the coefficient of mutual induction of a circle and a portion of a coaxial helix, beginning in the plane of the circle and of helical angle Θ. Then if M is the coefficient of mutual induction of the circle, and any portion of the helix for which the extreme points are determined by helical angles Θ_1 and Θ_2, we have

$$M = M_{\Theta_2} - M_{\Theta_1}.$$

It will therefore be sufficient to show how to calculate M_Θ for all values of Θ.

Let the equations to the circle and coaxial helix be

$$\left.\begin{aligned} y &= a \cos\theta \\ z &= a \sin\theta \\ x &= 0 \end{aligned}\right\} \qquad \left.\begin{aligned} y' &= A \cos\theta' \\ z' &= A \sin\theta' \\ x' &= p\theta' \end{aligned}\right\}.$$

Then it has been shown by the author* that M_Θ may be expressed by the following series which is convergent if $x < A - a$

$$M_\Theta = \Theta(A+a)c^2\Sigma(-1)^{m+1}\frac{1.3.5....(2m-1)}{2.4.6....2m}\frac{1}{2m+1}\left(\frac{x}{A+a}\right)^{2m}P_m,$$

where
$$c = \frac{2\sqrt{Aa}}{A+a}, \quad x = p\Theta,$$

$$P_m = \int_0^{\frac{\varpi}{2}} \frac{\cos 2\theta d\theta}{(1-c^2\sin^2\theta)^{\frac{2m+1}{2}}}.$$

* 'Phil. Mag.,' January, 1889.

§ 2. Putting $K_m = -\frac{1.3.5.\ldots(2m-1)}{2.4.6.\ldots 2m}\frac{1}{2m+1}\left(\frac{x}{A+a}\right)^{2m}P_m$,

so that $$M_\Theta = \Theta(A+a)c^2\Sigma(-1)^m K_m$$

it is now further proved that

$$K_{m+1} = \frac{2m(2m+1)}{(2m+2)(2m+3)}d^2\left\{K_m - \frac{(2m-1)(2m-3)}{2m\ .\ 2m}e^2K_{m-1}\right\}$$

where $$d^2 = \frac{1+c'^2}{c'^2}\left(\frac{x}{A+a}\right)^2$$

$$e^2 = \frac{1}{1+c'^2}\left(\frac{x}{A+a}\right)^2$$

$$c'^2 = 1-c^2.$$

Hence we see that the series is a recurring series. The relation above given between K_{m+1}, K_m, and K_{m-1} materially facilitates the calculation of successive terms of the series.

§ 3. If we put

$$\Sigma(-1)^m K_m = W$$

$$\Sigma(-1)^m m K_m = T$$

$$\Sigma(-1)^m \frac{m}{2m-1}K_m = V$$

and $$\frac{dM}{M} = q\frac{dA}{A} + r\frac{da}{a} + s\frac{dx}{x},$$

then $$q = \frac{1-s}{2} + \frac{T+2V}{deW}$$

$$r = \frac{1-s}{2} - \frac{T+2V}{deW}.$$

$$s = 2T/W.$$

§ 4. The following is a more general expression of M_Θ in terms of elliptic integrals applicable for all values of x:—

$$M_\Theta = \Theta(A+a)ck\left[\frac{F-E}{k^2} + \frac{c'^2}{c^2}(F-\Pi)\right]$$

where $$k = \frac{2\sqrt{Aa}}{\sqrt{(A+a)^2+x^2}}$$

$$F = \int_0^{\frac{\varpi}{2}} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$$

$$E = \int_0^{\frac{\varpi}{2}} \sqrt{1-k^2\sin^2\theta}\ .\ d\theta$$

$$\Pi = \int_0^{\frac{\varpi}{2}} \frac{d\theta}{(1-c^2\sin^2\theta)\sqrt{1-k^2\sin^2\theta}}.$$

The elliptic integral of the third kind Π is expressible by means of Legendre's formula* in terms of elliptic integrals of the first and second kind, complete and incomplete, and may so be calculated without difficulty.†

§ 5. Putting as before

$$\frac{dM}{M} = q\frac{dA}{A} + r\frac{da}{a} + s\frac{dx}{x},$$

it is shown that

$$q = \frac{A}{2aU}(F - c'\Pi)$$

$$r = \frac{a}{2AU}(F + c'\Pi)$$

$$s = -1 - \frac{1}{U}\left\{\left(1 - \frac{2}{k^2}\right)F + \frac{2}{k^2}E\right\}$$

where $$U = \frac{F-E}{k^2} + \frac{c'^2}{c^2}(F - \Pi) = \frac{\Theta(A+a)ck}{M}.$$

§ 6. The mutual potential energy of a circular current and a uniform coaxial circular cylindrical current sheet (the current lines being in planes at right angles to the axis) is identically the same as the mutual potential energy of the circular current and a coaxial helical current of the same radial and axial dimensions, beginning and ending in the ends of the sheet, if the current across a length of a generating line of the sheet equal to the pitch of the helix is equal to the helical current.

§ 7. The mutual potential energy of a helical current and a uniform coaxial circular cylindrical current sheet, or of two uniform coaxial circular cylindrical current sheets is expressible in terms of elliptic integrals.

§ 8. The electromagnetic force between a helical current and a uniform coaxial circular cylindrical current sheet is given by the formula

$$F = \gamma_h\gamma(M_2 - M_1)$$

where γ_h = the current in the helix,

γ = the current across unit length of a generating line of the sheet,

* 'Cayley,' "Elliptic Integrals," § 1̣3.

† 'Cayley,' chap. 13.

and M_1, M_2 are the coefficients of mutual induction of the circular ends of the sheet and the helix.

Hence the calculation of this force reduces itself to a double application of the formulæ for the coefficient of mutual induction of a circle and coaxial helix.

It is hoped that this may form a useful method of calculating the constant of current weighers designed to measure current in absolute units.

"A Note on some further Determinations of the Dielectric Constants of Organic Bodies and Electrolytes at very Low Temperatures."

By JAMES DEWAR, M.A., LL.D., F.R.S., Fullerian Professor of Chemistry in the Royal Institution, and J. A. FLEMING, M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London. Received October 28,—Read December 9, 1897.

In several previous communications* we have described the investigations made by us on the dielectric constants of various frozen organic bodies and electrolytes at very low temperatures. In these researches we employed a method for the measurement of the dielectric constant which consisted in charging and discharging a condenser, having the given body as dielectric, through a galvanometer 120 times in a second by means of a tuning-fork interrupter. During the past summer we have repeated some of these determinations and used a different method of measurement and a rather higher frequency. In the experiments here described we have adopted Nernst's method for the measurement of dielectric constants, using for this purpose the apparatus as arranged by Dr. Nernst which belongs to the Davy-Faraday Laboratory. The frequency of alternation employed was 350 or thereabouts, whereas in all our formerly described experiments with the galvanometer method it was 120.

The electrical details of the arrangement employed in Nernst's method are as follows:—A Wheatstone's bridge is formed (see diagram), two sides of which consist of variable resistances, R_1, R_2, which are usually liquid resistances contained in U-tubes. The other two sides of the bridge consist of two sliding condensers of variable capacity, C_1, C_2, which are shunted by adjustable liquid resistances, R_3, R_4. The bridge circuit contains a telephone, T, as detector. The alternating currents are furnished by an induction coil, I. An experimental condenser, X, the dielectric of which can be made

* See Fleming and Dewar, 'Roy. Soc. Proc.' (1897), vol. 61, pp. 2, 299, 316, 358, 368, and 381.